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DYNAMICAL METHODS TO ESTIMATE THE AGE OF ASTEROID FAMILIES

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Abstract

We present here some new results on the determination of the ages of asteroid families based on the analysis of motion of their members. Two independent methods have been used to assess the age of Veritas family, one making use of dynamical evolution of the stable family members, and the other where we analyze members with chaotic motion. In the former case we look for the refined age estimate by searching for convergence of angular elements to a narrow range of values acquired by the fragments immediately after the family formation event. In the latter case we compute the rate of chaotic diffusion of the resonant family members and estimate the time needed to spread from an initially compact distribution to the presently observed one.

Keywords: *Asteroids: families, resonances, chaotic diffusion*

1 Introduction

The ages of asteroid families, that is the epochs of their formation by collisional breakup of parent bodies, are of utmost importance for the study of dynamical and collisional evolution of asteroids. They serve to constrain the evolutionary models, to better understand physics of collisions and their outcomes, e.g. the initial velocity fields, to reveal the mechanical properties of asteroids, their internal composition and strength, to study the associated dust bands and their interaction with the Earth, etc.

Until quite recently the ages of asteroid families have been essentially unknown. The rough estimates, uncertain even at an order of magnitude level, have been inferred by using the indirect evidence such as cratering records or space weathering. Then **?** invented the method of *chaotic chronology* and applied it to estimate the age of Veritas family. They employed the fact that the two largest members of the family, asteroids (490) Veritas and (3542) Tanjiazhen, chaotically diffuse at a rate that will make them escape the region of the phase space occupied by the family in $\simeq 60$ My. Since both asteroids are at present still located well inside the family region, they concluded that the family cannot be older than 60 My, thus in fact establishing only an upper bound to its age. In the later studies along the same line (**??**), a more conservative upper bound to the age of the family of ~ 100 My has been proposed, but in the same time it has been pointed out that the true age might be significantly shorter.

A couple of new methods to estimate the age of asteroid families have been introduced by **?** and **?**. In the case of young families located in a region with no strong resonances, the family can be dated with the use of a method which consists in integrating backwards in time the equations of motion for the family members, until the orientation angles of their perturbed orbits converge to the values they had at the time of break-up. Using this latter method, **?** determined the age of the Karin family, the most recent break-up discovered so far in the asteroid belt which occurred ~ 5.8 My ago. Subsequently, **?** applied the same method also to the Veritas family and derived an estimate of the age of ~ 8.3 My. In the case of old families, the use is made of the distribution of the family members on the (H, a_{P}) plane, where H the absolute magnitude of a body and a_{P} the proper semi-major axis of its orbit. Due to the size dependent Yarkovsky non gravitational effect the family members drift in the semimajor axis, forming a characteristic V-shape. The age of the family can then be approximately found, by fitting the borders of the family with a theoretically expected curve depending on the physical parameters of the bodies (**?**).

In the following we will present some new results of the determination of the age of families, in particular an attempt to refine the existing estimate for Veritas family. These new results have been obtained by using the *modified chaotic chronology method* (**?**). The tools used for this research included numerical integration of orbits of family members and of fictitious asteroids for up to 100 My, computation of the time series of proper elements, of the Lyapunov times, of the coefficients of diffusion, etc.

2 Veritas family example

2.1 Procedures

We performed a large body of different analyzes to assess the distribution of Veritas family members in proper elements space, to identify the main dynamical groups, to analyze their dynamical behaviors and reveal the long-term effects of different resonant multiplets and the corresponding evolutionary time scales, and to study the cases of diffusive and non-diffusive chaos. Eventually we introduced a modified chaotic chronology method to estimate the age of this family. The results of this work are presented in full detail in **?**; here we shall briefly sketch only the most important ones.

We first identified the members of the Veritas family by using the Hierarchical Clustering Method (**?**) and finding a total of 180 members for the value of the velocity cut-off $v_\mathrm{c} = 40\,\mathrm{m\,s^{-1}}$. For each real family member and for a set of 400 fictitious objects we computed the Lyapunov times T_L by integrating the variational equations. Next we studied the long-term evolution of the proper orbital elements of family members for 100 My. We made two integrations: first, all 180 real objects were integrated for 10 My, then the integration has been extended to 100 My for a sub-group of representative objects. The equations of motion were integrated by means of the public domain *Orbit 9* software package[1]. Using the analytical theory by **?**, we subsequently computed the corresponding time series of proper elements for all integrated bodies. A 100 My integration backwards in time, followed also by a computation of the corresponding time series of proper elements, was performed for the members of the family with regular orbits.

For the resonant family members, we also calculated the mean squared displacement, $\langle(\Delta J_\mathrm{k})^2\rangle$, of the two action variables related to e_P and i_P, the average being taken over the corresponding set of family members. In terms of a simple diffusion approximation, the mean squared displacement in action grows linearly with time, at a characteristic rate called *the diffusion coefficient*. The diffusion coefficient in each action, $\mathcal{D}(J_\mathrm{k})$, was computed as the least-squares-fit slope of the $\langle(\Delta J_\mathrm{k})^2\rangle\,(t)$ curve. Finally, a simple random-walk model has been used to describe the evolution of chaotic family members, where, at every Δt, each body undergoes a random 'jump', whose length in J_1 and J_2 is given by a 2-D Gaussian distribution. The values of $\mathcal{D}(J_1)$ and $\mathcal{D}(J_2)$ then correspond to the standard deviation of the projections of the probability density, along the J_1 and J_2 axis, respectively.

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[1] Available from the *AstDyS* service at http://hamilton.dm.unipi.it/astdys

2.2 Results

The Veritas family members exhibit a complex distribution in the space of proper elements. In particular, the $(a_{\rm P}, e_{\rm P})$ distribution reveals that the family can be decomposed into at least four dynamically distinct groups, arranged along the $a_{\rm P}$ values spanned by its members. Bodies clustered around $a_{\rm P} = 3.168$ AU (group B) and around $a_{\rm P} = 3.174$ AU (group A) have significantly larger spread in $e_{\rm P}$, than the rest of the family. The low a side of the family (group R_1), mostly composed of bodies on regular orbits (**?**), fits well inside the elliptical equivelocity curve computed using the equations of Gauss (**?**), while groups A and B are cutting through this ellipse, producing two 'finger-like' features. If we assume that the family borders immediately after the break-up can be approximated by an equivelocity ellipse, then the initial spread in eccentricity for group-A and group-B objects should have been much smaller than observed at present. We, however, know that these 'finger-like' features are associated with the strong three-body mean motion resonances (**???**) among Jupiter, Saturn and the asteroid. The remaining component of the family (group R_2) is located between the two dispersed groups and has again a small spread in $e_{\rm P}$.

In order to analyze the dynamical behavior of the main groups we first carried out an analyzis of the chaotic zones and resonances involved. We found that there are two broad chaotic bands cutting through the family, characterized by two different values of $T_{\rm L}$: group-A bodies, including (490) Veritas itself, reside in the $T_{\rm L} \approx 10^4$ yr chaotic band, while most group-B bodies are located in the $T_{\rm L} \approx 3 \cdot 10^4$ yr band. The rest of group-B asteroids as well as R_2 bodies are located between these two main chaotic bands. Chaotic sidebands are also observed on both sides of the $T_{\rm L} = 10^4$ yr band, as well as some less important features. While it is well known that the group-A chaotic band is associated with the (5,−2,−2) Jupiter-Saturn-asteroid mean motion resonance (**?**), the $T_{\rm L} = 3 \cdot 10^4$ yr band is associated with the (3,3,−2) resonance (**?**); also asteroid (37005) moves in the vicinity of the (7,−7,−2) resonance. These two latter resonances are due to frequency combinations of the main (5,−2,−2) resonance and the 2/5 near resonance of Jupiter and Saturn.

Group R_1 consists entirely of asteroids on regular orbits, their proper elements being stable over a 100 My time interval. A somewhat different behavior is observed for the members of the R_2 group, as well as for the members of group B located outside the (3,3,−2) chaotic band. The nearby resonances induce long-periodic, small amplitude perturbations, but these perturbations do not build-up, at least within 100 My. Thus R_2 and non-chaotic asteroids of group-B also have very stable proper elements over a 100 My time span and their long-term behavior is very similar to that of R_1 bodies. In our study of the past evolution of regular family members we, however, considered R_1 bodies only, since they are less affected by the nearby resonances.

By examining more closely the motion inside the two main chaotic bands, we found that the group-A bodies diffuse very efficiently in e_P and $\sin i_\mathrm{P}$. As inferred from our integrations, already after 10 My they cover a phase-space region that is about twice as big as the initial one. The behavior of group-B asteroids is quite different as their evolutionary traces cover practically the same region as the one occupied by their present-day distribution, and no macroscopic diffusion is observed. This might appear as somewhat surprising in view of not so very different Lyapunov times for the two groups. However, the instabilities of proper elements are related to the resonances involved, so that different resonances can have similar values of T_L in their chaotic layers, but very different long-term stability properties (hence the notion *stable chaos*; see **?**). We actually found several group-B bodies, whose mean semi-major axis changes in time, the asteroid 'jumping' from one component of the resonance multiplet to another, but this type of chaotic motion does not seem to produce macroscopic diffusion in e_P and $\sin i_\mathrm{P}$.

Using the 10 My integrations of the real bodies, we computed the diffusion coefficients in J_1 and J_2 for both chaotic groups. The mean squared displacement in both actions has been found to be a linear function of time and the corresponding values of the diffusion coefficient were $\mathcal{D}(J_1) = 1.1 \cdot 10^{-14}$ yr^{-1} and $\mathcal{D}(J_2) = 1.4 \cdot 10^{-14}$ yr^{-1}. On the contrary, for the chaotic group-B asteroids, we found a much smaller value for $\mathcal{D}(J_1) = 9.6 \cdot 10^{-17}$ yr^{-1} and a much smaller value for $\mathcal{D}(J_2) = 1.9 \cdot 10^{-18}$ yr^{-1}. Thus, chaotic motion is strongly diffusive for group A, while it is almost non-diffusive for group B.

Finally, we derived two independent estimates of the age of Veritas family, one making use of dynamical evolution of the stable family members, and the other based on the members with chaotic motion. In the former case we integrated backwards in time the orbits of the 50 family members of R_1 for 10 My and calculated their proper elements time series. Following **?**, we computed the mean difference of nodal longitudes of all bodies, $\langle \Delta\Omega \rangle$, with respect to the largest member of the family with a stable orbit, asteroid (1086) Nata, as a function of time. A clustering of the nodes within 30° was found at $t = -8.3$ My. Thus, we have confirmed the result obtained by **?**, but our results show a more tight clustering of the nodes (within 30°) mainly because we excluded from the calculation all non-chaotic B and R_2 family members, and because we made use of proper elements, thus reducing the effects of short periodic variations.

An independent estimate of the age of the family can be obtained, using a modified chaotic chronology method for group A asteroids. A simple random-walk model can be used to describe their evolution, as described in Section 1. Starting from an initial distribution extended as suggested by the equivelocity ellipses, we took a number of snapshots of the evolution of a fictitious initial distribution of 400 (5,−2,−2)-resonant bodies in the $(e_{\rm P}, \sin i_{\rm P})$ space, according to our random-walk model, and we superimpose it on the present-day distribution of group-A family members.

The fictitious objects are spreading diffusively in action space, the variance of the distribution growing linearly with time. The box in a 2-D phase space representing the observed distribution is filled by our random-walkers within less than 10 My; even several particles have leaked out of the box, but still more than 67% of the particles is well contained within the box in both variables. Next we checked the area of the plane covered by the real group-A members, when the latter are propagated for 10 My into the future. At $t = 20$ My, the random-walkers cover an area, which has approximately the same extent as the one covered by the future images of real group-A members. The above results confirm that a simple random-walk approach can be used to obtain an estimate for the age of group A, and, in fact, they indicate that the age of this group must be ~ 10 My.

Of course, the diffusion zone spanned by group-A asteroids does not have the simple elliptical shape of the region covered by the random-walkers. In particular it has three 'tails', two at small and one at high eccentricities, while most of the bodies seem to occupy a more compact region. This suggests that, as the resonant bodies spread, they approach phase-space regions, characterized by different transport properties: the diffusion coefficient may vary significantly, with respect to the region occupied by the present-day distribution of group-A bodies, or transport may deviate significantly from normal diffusion, due to a complex phase-space topology. In either case, the normal diffusion is only an approximation of the real transport process. However, the simple diffusion approximation seems to provide a good estimate for the transport time scale.

In other words, the above results show that, even though the evolution of the distribution function of group-A bodies cannot be assumed to follow Fick's law (see Eq. (1) in **?**) for all times, their *current* distribution can be viewed as the evolution of a narrow initial distribution in J_1, whose variance grows with time as $\sigma^2(x) = \mathcal{D}/2 \cdot t$. Using the obtained values of $\mathcal{D}(J_1)$ and of $\sigma(J_1) = 2.2 \cdot 10^{-4}$ taken from the data, we estimate the age of group A to be $\tau_1 = (9.0 \pm 1.3)$ My. This value agrees satisfactorily with the 8.3 My age-estimate of the regular component of the family, taking into account the approximative nature of our assumptions. A similar calculation can be performed for the J_2 distribution. The corresponding values are $\sigma(J_2) = 3.6 \cdot 10^{-4}$ and $\tau_2 = (18.6 \pm 2.7)$ My. The reason for which τ_1 is closer to the value obtained from the analysis of the regular bodies is that the initial J_1 distribution is much closer to a delta function, which is the expected distribution of family members immediately after break-up, than the J_2 distribution.

3 Conclusion

We have shown in this paper that the methods to determine the age of asteroid families improved in the recent years to the point that we can nowadays estimate the age of a number of asteroid families quite accurately, providing they comprise members having either very stable, or strongly (diffusively) chaotic orbits. These dynamical methods have been applied in a number of cases (Veritas, Karin, Eos), of which we have described here the case of Veritas. The results of the application of two independent methods presented here agree fairly well, and this indicates that the family of Veritas has most probably been created as a whole in a single break-up event. It is a very young family, and indeed the youngest known family in the main asteroid belt originated from the parent body as large as 150 km in diameter.

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References

Knežević, Z. 1999, in: *Evolution and source regions of asteroids and comets. Proc. IAU Coll. 173*, J. Svoren, E.M. Pittich and H. Rickman (eds.), Astron. Inst. Slovak Acad. Sci., Tatranska Lomnica, p. 153

Knežević, Z., Pavlović, R. 2002, Earth, Moon, Planets, **88**, 155

Knežević, Z., Tsiganis, K., and Varvoglis, H. 2002. in: *Proceedings of the Conference Asteroids, Comets, Meteors (ACM 2002)*, B. Warmbein (ed.), (Noordwijk: ESA-SP-500), p. 335

Milani, A., and Nobili, A.M. 1992, Nature, **357**, 569

Milani, A., Farinella, P. 1994, Nature, **370**, 40

Milani, A., and Knežević, Z., 1994. Icarus **107**, 219

Morbidelli, A. Zappalà, V., Moons, M., Cellino, A., and Gonczi, R. 1995. Icarus, **118**, 132

Nesvorný, D., and Morbidelli, A., 1998. AJ, **116**, 3029

Nesvorný, D., Bottke, W.F. Jr., Dones, L., Levison, H. F. 2002, Nature, **417**, 720

Nesvorný, D., Bottke, W.F., Levison, H.F., Dones, L., 2003, ApJ, **591**, 486

Tsiganis, K., Knežević, Z. Varvoglis, H. 2006, Icarus, submitted.

Vokrouhlický, D., Brož, M., Bottke, W.F., Nesvorný, D., Morbidelli, A. 2005, in: *Dynamics of Populations of Planetary Systems, Proceedings IAU Coll. 197*, Z. Knežević and A. Milani (eds.) Cambridge University Press, p. 145

Zappalà, V., Bendjoya, Ph., Cellino, A., Farinella, P., and Froeschlé, C., 1995. Icarus, **116**, 291